

ECS 455 Chapter 1

Introduction & Review

Radio

1.3 Wireless Channel (Part 1)

{ noise
interference } random, time-varying

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Tuesday 9:30-10:30

Tuesday 13:30-14:30

Thursday 13:30-14:30

Wireless Channel

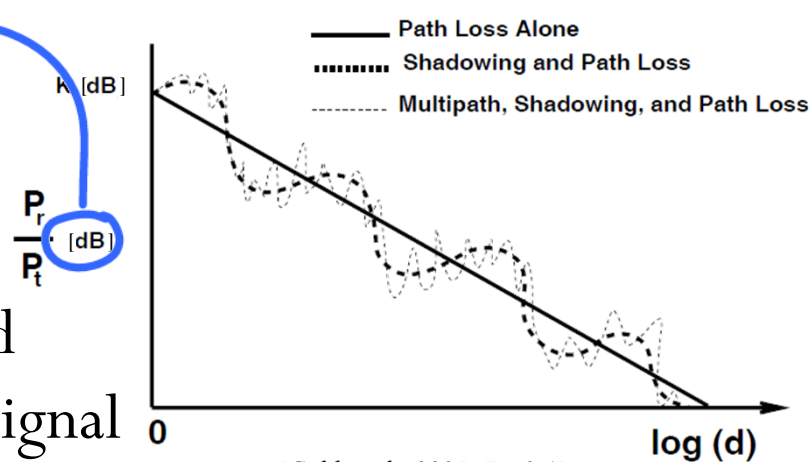
- **Large-scale** propagation effects

1. Path loss
2. Shadowing

3. • **Small-scale** propagation effects

- Variation due to the constructive and destructive addition of **multipath** signal components.
- Occur over very **short distances**, on the order of the signal **wavelength**.

$$10 \log_{10} \frac{P_r}{P_t}$$



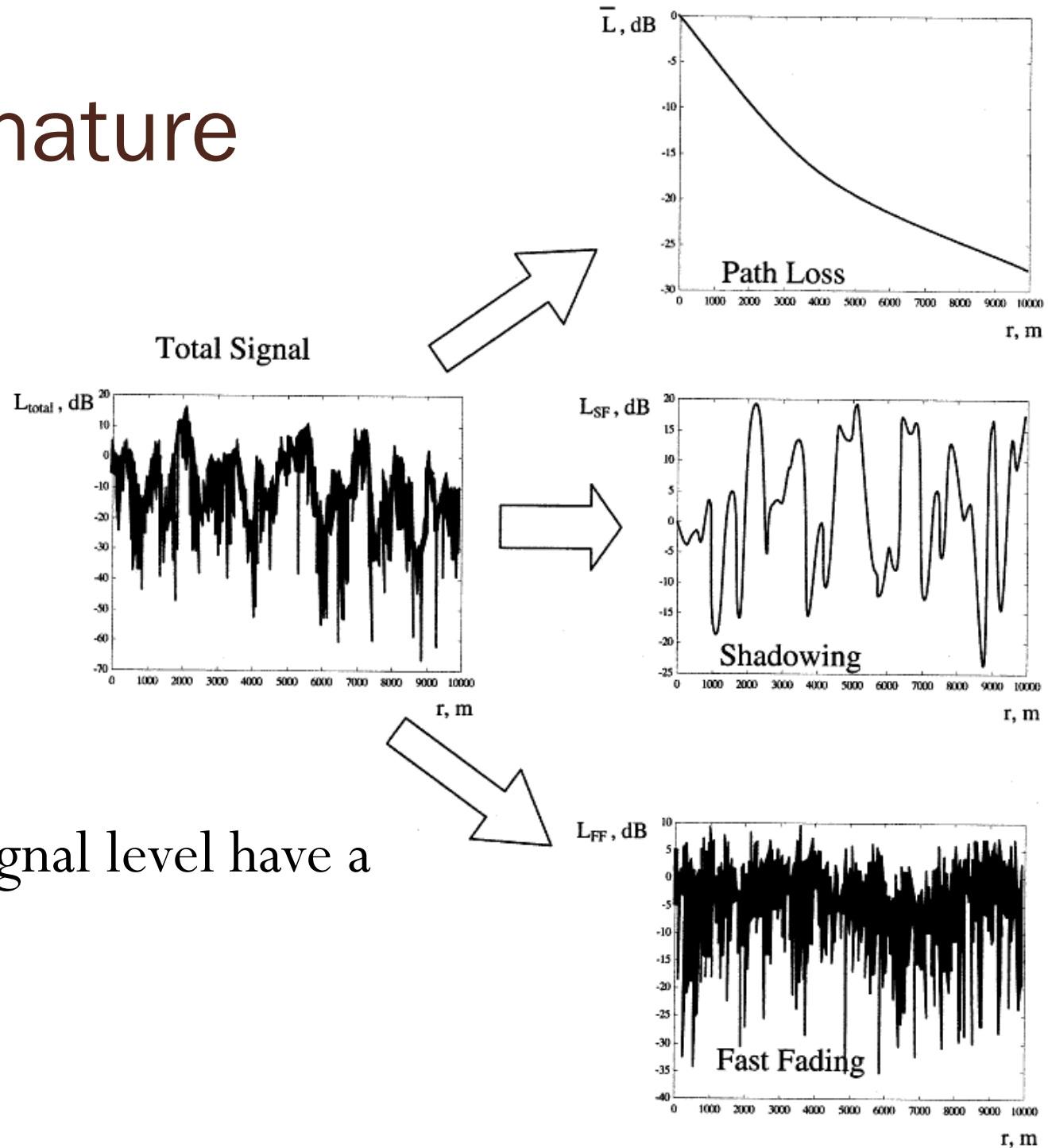
[Goldsmith, 2005, Fig 2.1]

$$\lambda = \frac{c}{f} \leftarrow \approx 3 \times 10^8 \text{ [m/s]}$$

3×10^9

$$f = 3 \text{ GHz} \rightarrow \lambda = 0.1 \text{ m}$$

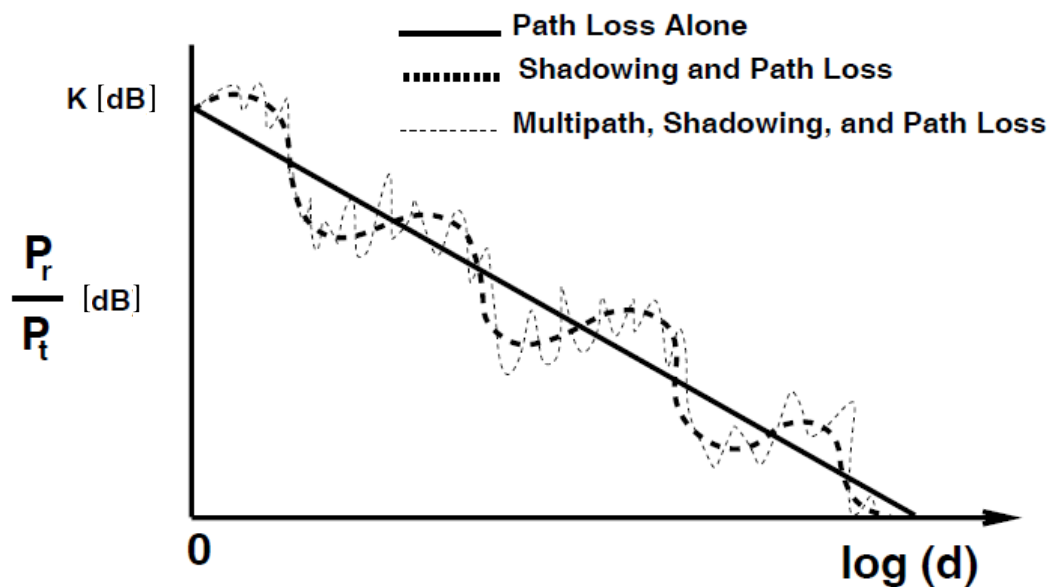
Triple nature



Variations of signal level have a triple nature.

Path loss

- Caused by
 - dissipation of the power radiated by the transmitter
 - effects of the propagation channel
- Models generally assume that it is the same at a given transmit-receive distance.
- Variation occurs over **large distances** (100-1000 m)



[Goldsmith, 2005, Fig 2.1]

Path Loss (PL)

$$P_L = \frac{\text{Transmitted power}}{\text{Average received power}} = \frac{P_t}{P_r}$$

Averaged over any random variations

- **Free-Space** Path Loss Model:

$$\frac{P_r}{P_t} \propto \frac{1}{d^2}$$

- P_r falls off inversely proportional to the square of the distance d between the Tx and Rx antennas.

- **Simplified** Path Loss Model:

$$\frac{P_r}{P_t} = K \left(\frac{d_0}{d} \right)^\gamma$$

To be discussed

(Path loss of the free-space model)

Friis Equation (Free-Space PL)

- One of the most fundamental equations in antenna theory

1 for non-directional antennas

$$\frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx} G_{Rx}} \lambda}{4\pi d} \right)^2 = \left(\frac{\sqrt{G_{Tx} G_{Rx}} c}{4\pi df} \right)^2$$

- Lose more power at higher frequencies.

0.7 GHz → 2.4 GHz → 5 GHz → 60 GHz

10.7 dB loss

$$20 \log_{10} \frac{2.4}{0.7}$$

6.4 dB loss

$$20 \log_{10} \frac{5}{2.4}$$

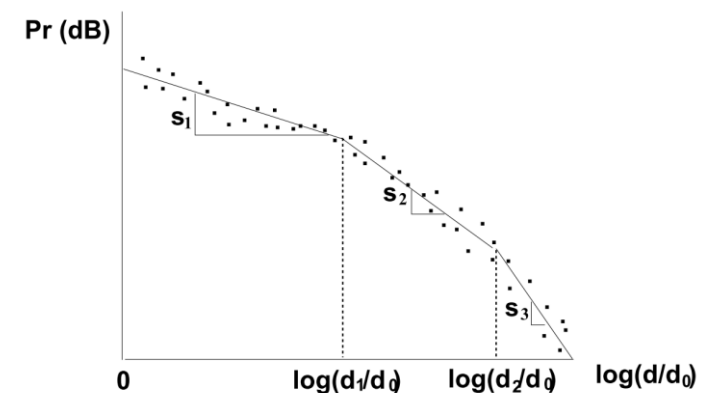
21.6 dB loss

$$20 \log_{10} \frac{60}{5}$$

- Some of these losses can be offset by reducing the maximum operating range.
 - The remaining loss must be compensated for by increasing the antenna gain.

More Path Loss Models

- Analytical models
 - Maxwell's equations
 - Ray tracing
 - Empirical models: Developed to predict path loss in typical environment.
 - Okumura
 - Hata
 - COST 231
 - by EURO-COST (EUROpean COoperative for Scientific and Technical research)
 - Piecewise Linear (Multi-Slope) Model
 - Tradeoff: Simplified Path Loss Model
- Prohibitive (complex, impractical)
Need to know/specify “almost everything” about the environment.



Simplified Path Loss Model

$$\frac{P_r}{P_t} = K \left(\frac{d_0}{d} \right)^\gamma$$

$$10 \log_{10} \frac{P_r}{P_t} \text{ [dB]} = \left(10 \log_{10} K d_0^\gamma \right) - 10\gamma \log_{10} d$$

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- K is a unitless constant which depends on the antenna characteristics and the average channel attenuation
 - $\left(\frac{\lambda}{4\pi d_0} \right)^2$ for free-space path gain at distance d_0 assuming omnidirectional antennas
- d_0 is a reference distance for the antenna far-field
 - Typically 1-10 m indoors and 10-100 m outdoors.
- γ is the **path loss exponent**.

(Near-field has scattering phenomena.)

Path Loss Exponent γ

- 2 in free-space model
- 4 in two-ray model
[Goldsmith, 2005, eq. 2.17]
- Cellular: 3.5 – 4.5
[Myung and Goodman, 2008 , p 17]
- Larger @ higher freq.
- Lower @ higher antenna heights

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

Indoor Attenuation Factors

- Building penetration loss: 8-20 dB (better if behind windows)
- Attenuation between floors
 - @ 900 MHz
 - 10-20 dB when the Tx and Rx are separated by a single floor
 - 6-10 dB per floor for the next three subsequent floors
 - A few dB per floor for more than four floors
 - Typically worse at higher frequency.
- Attenuation across floors

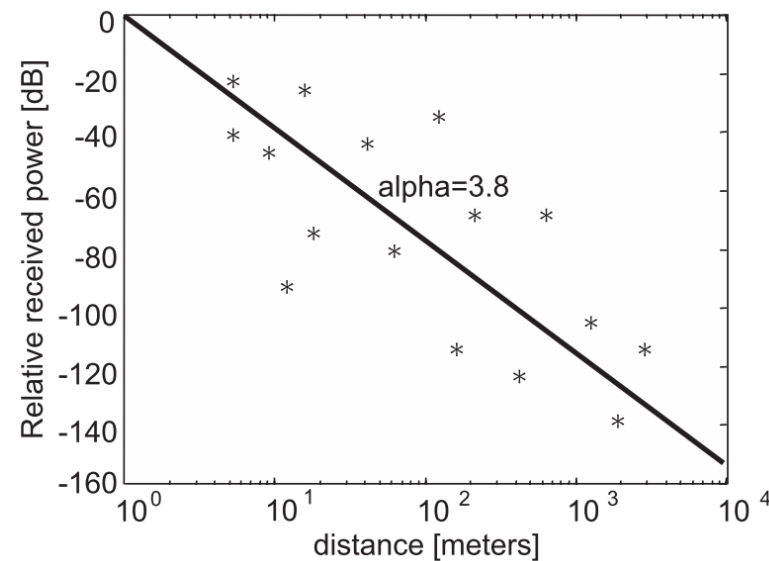
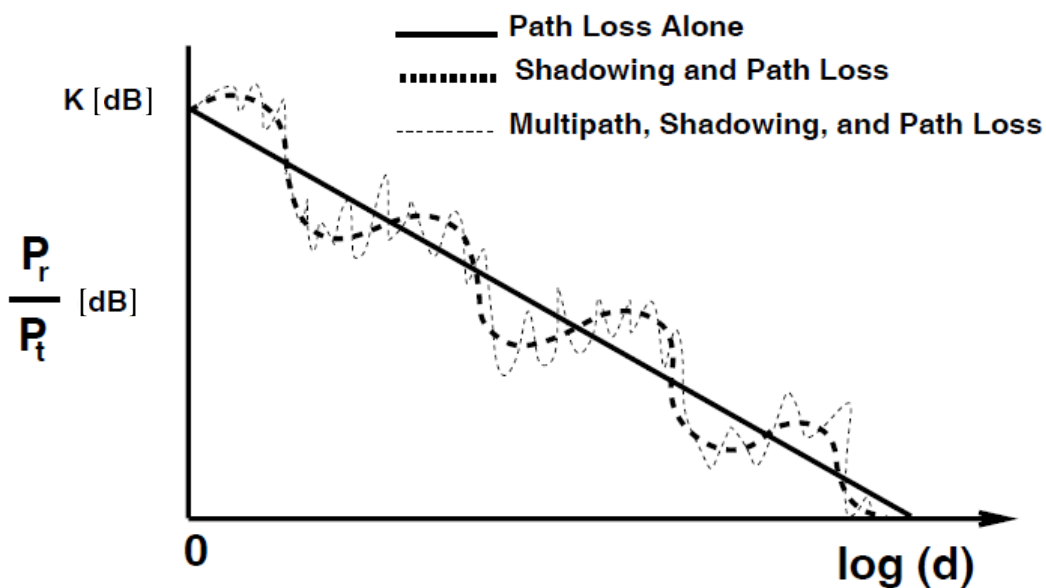
Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

[Goldsmith, 2005, Sec. 2.5.5]

Shadowing (or Shadow Fading)

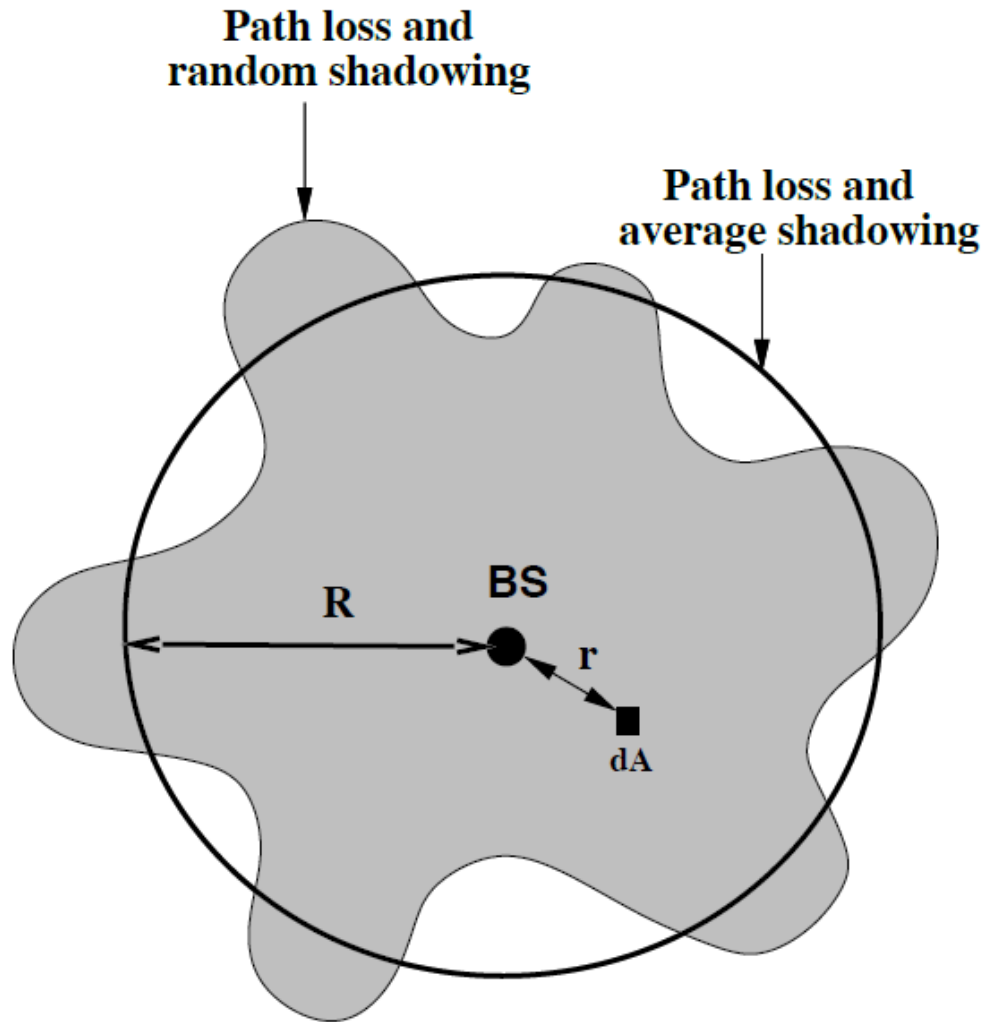
- Additional attenuation caused by **obstacles** (large objects such as buildings and hills) between the transmitter and receiver.
 - Think: cloud blocking sunlight
- Attenuate signal power through absorption, reflection, scattering, and diffraction.
- Variation occurs over distances proportional to the length of the obstructing object (**10-100 m** in outdoor environments and less in indoor environments).

[Goldsmith, 2005, Fig 2.1]



[Myung and Goodman, 2008, Fig 2.1]

Contours of Constant Received Power



[Goldsmith, 2005, Fig 2.10]

Log-normal shadowing

- Random variation due to blockage from objects in the signal path and changes in reflecting surfaces and scattering objects
→ random variations of the received power at a given distance

$$10 \log_{10} \frac{P_t}{P_r} \sim \mathcal{N}(\mu, \sigma^2)$$

4 – 13 dB with higher values in urban areas and lower ones in flat rural environments.

in dB

- This model has been confirmed empirically to accurately model the variation in received power in both outdoor and indoor radio propagation environments.

[Erceg et al, 1999] and [Ghassemzadeh et al, 2003]

Log-normal shadowing (motivation)

- Location, size, dielectric properties of the blocking objects as well as the changes in reflecting surfaces and scattering objects that cause the random attenuation are generally unknown
⇒ statistical models must be used to characterize this attenuation.
- Assume a large number of shadowing objects between the transmitter and receiver

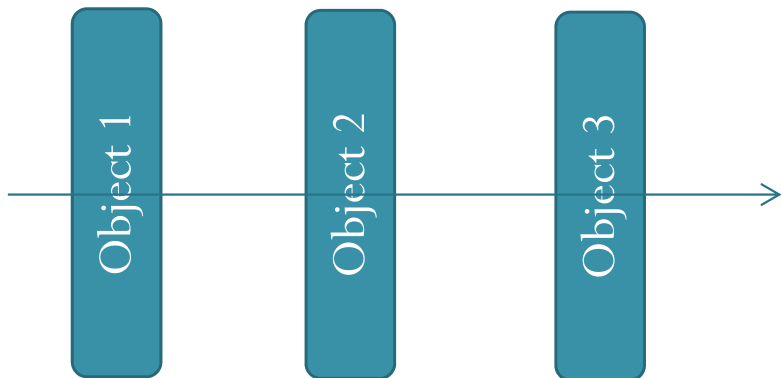
Without the objects, the attenuation factor is $K \left(\frac{d_0}{d} \right)^\gamma$.

Each object introduces extra power loss factor of α_i .

So,

$$\frac{P_r}{P_t} = K \left(\frac{d_0}{d} \right)^\gamma \prod_i \alpha_i$$

$$10 \log_{10} \frac{P_r}{P_t} = 10 \log_{10} K \left(\frac{d_0}{d} \right)^\gamma + \underbrace{\sum_i 10 \log_{10} \alpha_i}_{\text{By CLT, this is approximately Gaussian}}$$



PDF of Lognormal RV

- Consider a random variable

$$R = \frac{P_t}{P_r}$$

- Suppose

$$10 \log_{10} R \sim \mathcal{N}(\mu, \sigma^2)$$

Here, it should be clear why the unit of σ is in dB.

- Then,

$$f_R(r) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} \frac{10}{\ln 10} \frac{1}{r} e^{-\frac{1}{2} \left(\frac{(10 \log r) - \mu}{\sigma} \right)^2}, & r > 0 \\ 0, & \text{otherwise.} \end{cases}$$

For typical cellular environment, σ is in the range of 5-12 dB.
[Proakis and Salehi, 2007, p 843]

PDF of Lognormal RV (Proof)

Suppose $c \log_b Y \sim \mathcal{N}(\mu, \sigma^2)$.

Let $X = c \log_b Y$. Note that $X = c \log_b Y = \frac{c}{\ln b} \ln(Y) = k \ln(Y)$.

Then, $Y = e^{\frac{X}{k}}$ where $k = \frac{c}{\ln b}$.

Recall, from ECS315 that to find the pdf of $Y = g(X)$ from the pdf of X , we first find the cdf of Y and then differentiate to get its pdf:

$$F_Y(y) = P[Y \leq y] = P\left[e^{\frac{X}{k}} \leq y\right] = P[X \leq k \ln(y)] = F_X(k \ln(y)).$$

$$f_Y(y) = \frac{d}{dy} F_X(k \ln(y)) = \frac{k}{y} f_X(k \ln(y)) = \frac{1}{\sqrt{2\pi\sigma}} \frac{k}{y} e^{-\frac{1}{2}\left(\frac{k \ln(y) - \mu}{\sigma}\right)^2}.$$

PDF of Lognormal RV (Proof)

Suppose $c \log_b Y \sim \mathcal{N}(\mu, \sigma^2)$.

Let $X = c \log_b Y$. Note that $X = c \log_b Y = \frac{c}{\ln b} \ln(Y) = k \ln(Y)$.

Then, $Y = e^{\frac{X}{k}}$ where $k = \frac{c}{\ln b}$.

Alternatively, to find the pdf of $Y = g(X)$ from the pdf of X , when g is monotone, we may use the formula:

$$f_X(x) |dx| = f_Y(y) |dy| \longrightarrow f_Y(y) = \left| \frac{dx}{dy} \right| f_X(x)$$

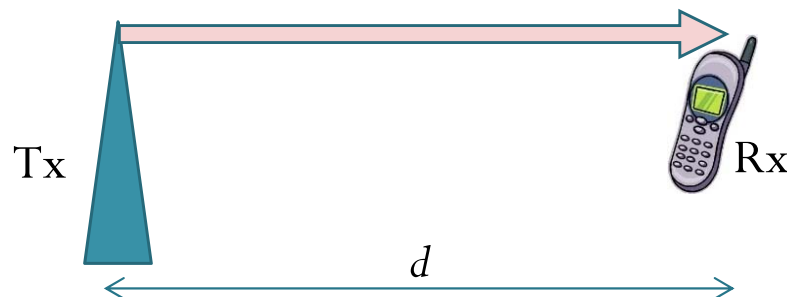
This gives $f_Y(y) = \frac{k}{y} f_X(c \log_b y)$ (same as what we found earlier).

Ray tracing (a prelude)

- Approximate the solution of Maxwell's equations
 - Approximate the propagation of electromagnetic waves by representing the wavefronts as simple **particles**.
 - Thus, the reflection, diffraction, and scattering effects on the wavefront are approximated using **simple geometric equations** instead of Maxwell's more complex wave equations.
- Assumption: the received waveform can be approximated by the sum of the free space wave from the transmitter plus the reflected free space waves from each of the reflecting obstacles.

$$x(t) = \sqrt{2P_t} \cos(2\pi f_c t)$$

$$y(t) = ?$$



Energy and Power

- Consider a signal $g(t)$.
- Total (normalized) **energy**:

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \lim_{T \rightarrow \infty} \int_{-T}^T |g(t)|^2 dt \stackrel{\text{Parseval's Theorem}}{=} \int_{-\infty}^{\infty} |G(f)|^2 df.$$

$$\Psi_g(f) = |G(f)|^2$$

ESD: Energy Spectral Density

- Average (normalized) **power**:

$$P_g = \left\langle |g(t)|^2 \right\rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |g(t)|^2 dt.$$

Three Important Facts

- If $g(t)$ is periodic with period T_0 , then $P_g = \frac{1}{T_0} \int_{T_0} |g(t)|^2 dt$
- If $g(t)$ can be expressed as $\sum_{k=1}^n c_k e^{j2\pi f_k t}$ where the f_k are distinct, then $P_g = \sum_{k=1}^n |c_k|^2$.

- Ex

- Suppose that $g(t)$ can be expressed as $a(t)\cos(2\pi f_c t + \phi)$ and that $A(f - f_c)$ and $A(f + f_c)$ do not overlap. Then

$$P_g = \frac{1}{2} P_a$$

Power Calculation (More Examples)

$$g(t) = \sum_k a_k(t) \cos(2\pi f_k t + \phi_k)$$

$$P_g = \frac{1}{2} \sum_k P_{a_k}$$

Assume the $A_k(f \pm f_k)$'s do not overlap.

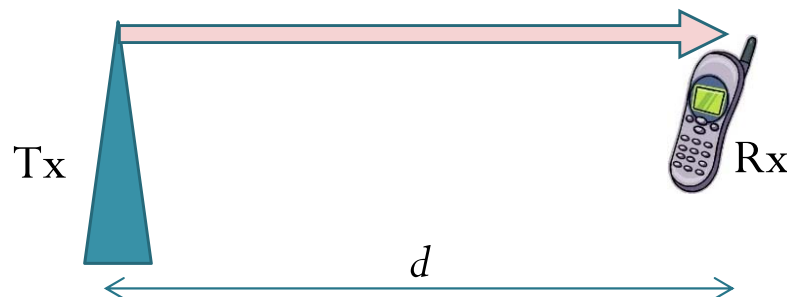
$$g(t) = a_1 \cos(2\pi f_c t + \phi_1) + a_2 \cos(2\pi f_c t + \phi_2)$$

$$P_g = \frac{1}{2} a_1^2 + \frac{1}{2} a_2^2 + a_1 a_2 \cos(\phi_2 - \phi_1)$$

Ray tracing

- Approximate the solution of Maxwell's equations
 - Approximate the propagation of electromagnetic waves by representing the wavefronts as simple **particles**.
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- Assumption: the received waveform can be approximated by the sum of the free space wave from the transmitter plus the reflected free space waves from each of the reflecting obstacles.

$$x(t) = \sqrt{2P_t} \cos(2\pi f_c t) \quad y(t) = \frac{\alpha}{d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{d}{c}\right)\right)$$

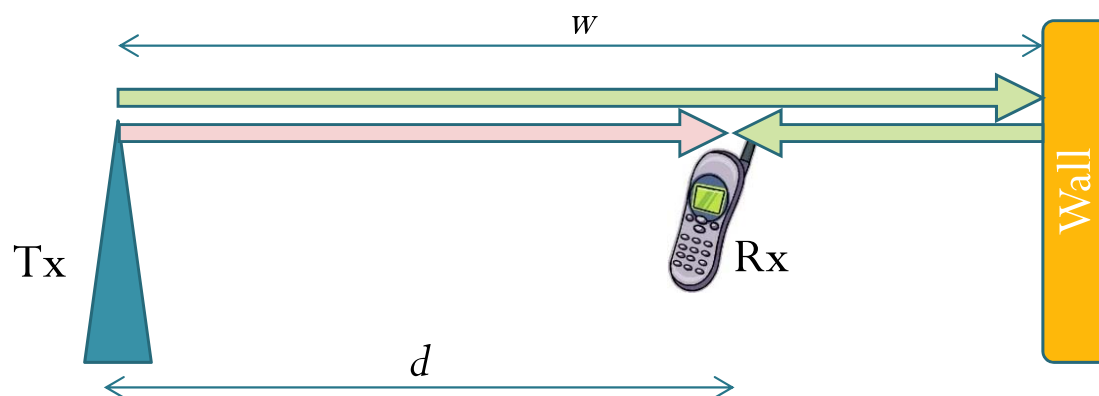


From Friis equation,

$$\alpha = \frac{\sqrt{G_{Tx} G_{Rx}} \lambda}{4\pi}$$

Ex. One reflecting wall (1/4)

- There is a fixed antenna transmitting the sinusoid $x(t)$, a fixed receive antenna, and a single perfectly reflecting large fixed wall.
- Assume that the wall is very large, the reflected wave at a given point is the same (except for a sign change) as the free space wave that would exist on the opposite side of the wall if the wall were not present



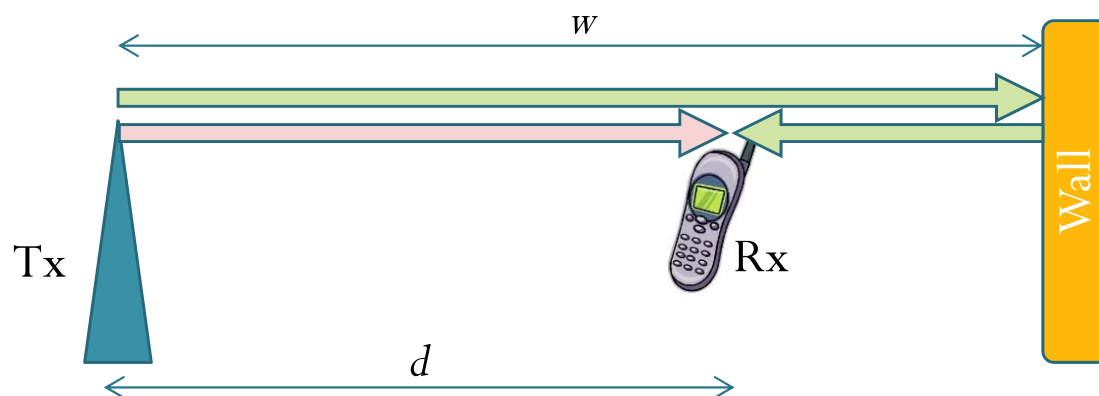
Ex. One reflecting wall (2/4)

$$x(t) = \sqrt{2P_t} \cos(2\pi f_c t)$$

$$y(t) = \frac{\alpha}{d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{d}{c}\right)\right) - \frac{\alpha}{2w-d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{2w-d}{c}\right)\right)$$

$$= \frac{\alpha}{d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{d}{c}\right)\right) - \frac{\alpha}{2w-d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{2w-d}{c}\right)\right)$$

$$= \frac{\alpha}{d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{d}{c}\right)\right) + \frac{\alpha}{2w-d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{2w-d}{c}\right)\right)$$



Ex. One reflecting wall (3/4)

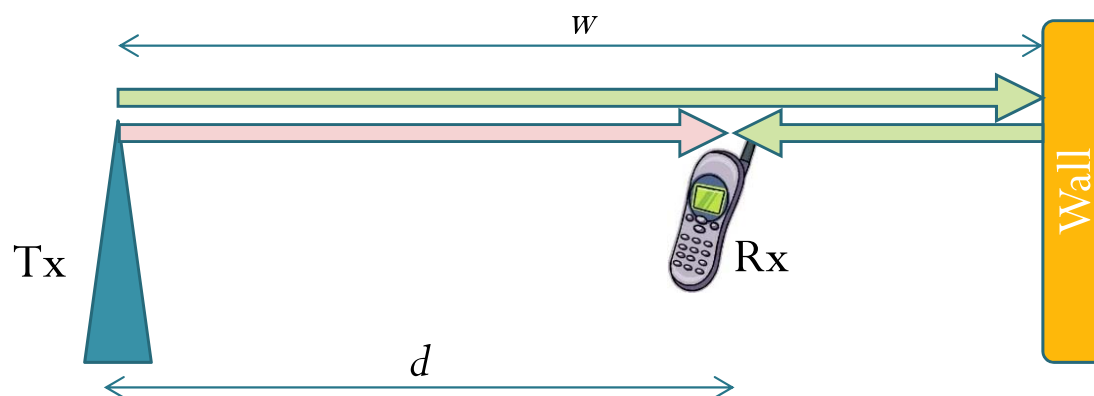
$$y(t) = \frac{\alpha}{d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{d}{c}\right)\right) + \frac{\alpha}{2w-d} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{2w-d}{c}\right) - \pi\right)$$

$$P_y = P_t \left(\left(\frac{\alpha}{d}\right)^2 + \left(\frac{\alpha}{2w-d}\right)^2 + 2 \frac{\alpha^2}{d(2w-d)} \cos(\Delta\phi) \right)$$

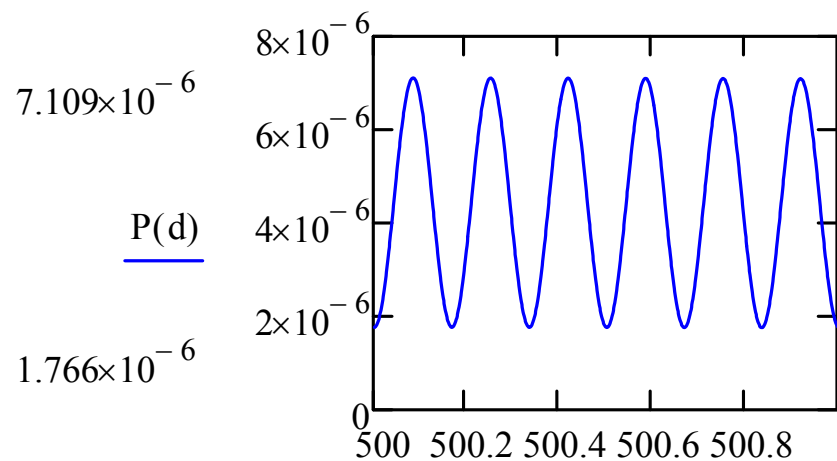
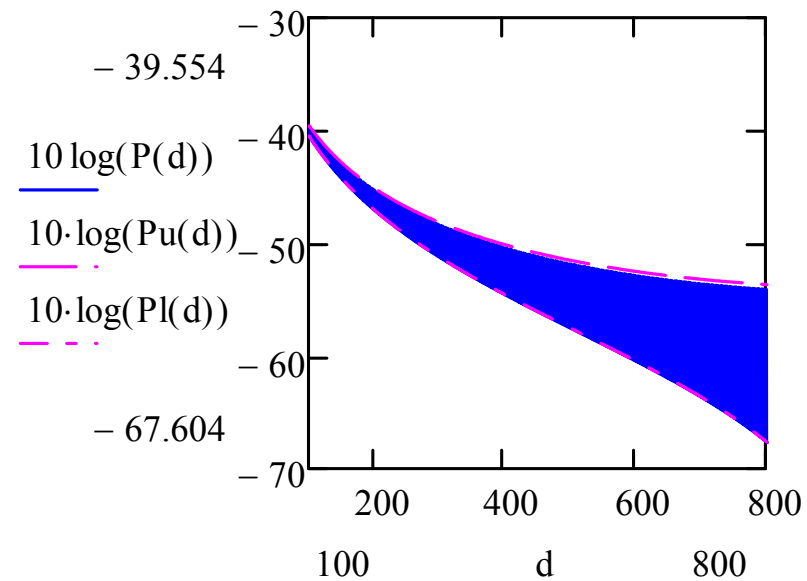
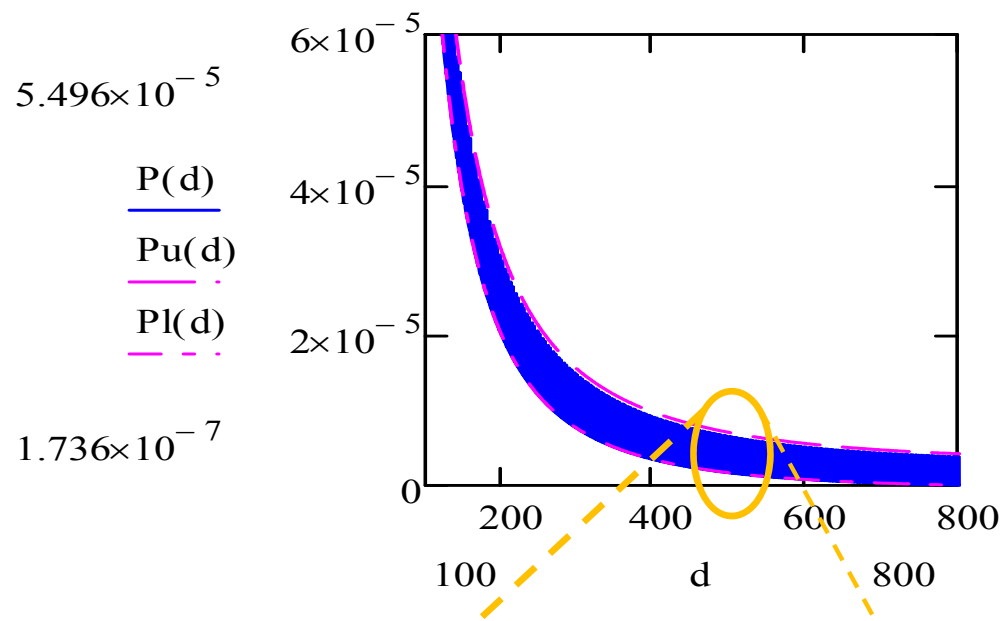
$$\Delta\phi = 2\pi f_c \frac{2w-2d}{c} + \pi = 2\pi \frac{1}{\lambda/2} (w-d) + \pi$$



form constructive
and destructive
interference
pattern



Ex. One reflecting wall (4/4)



$$f = 900 \text{ MHz}$$

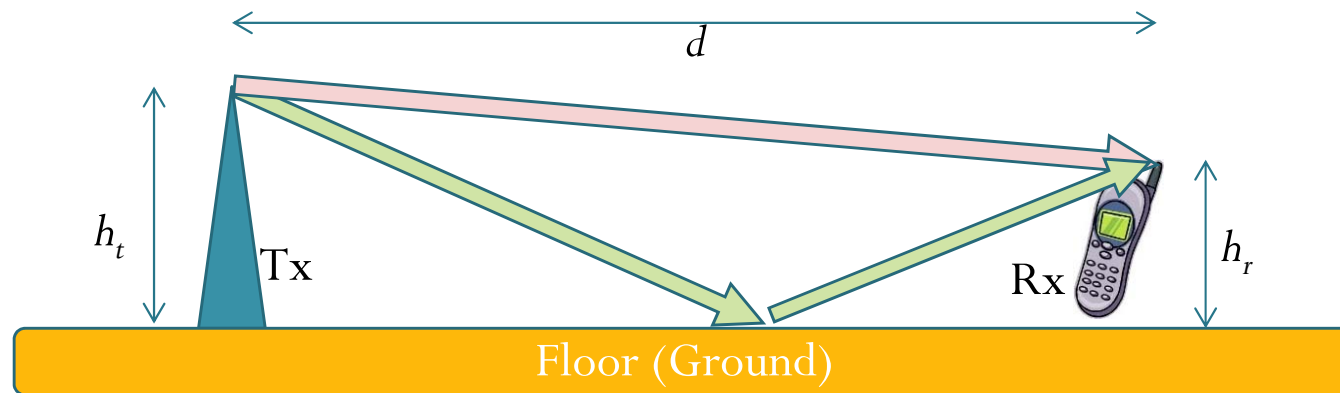
$$w = 1 \text{ km}$$

Ex. Two-Ray Model

$$\text{Delay spread} = \frac{r_2}{c} - \frac{r_1}{c}$$

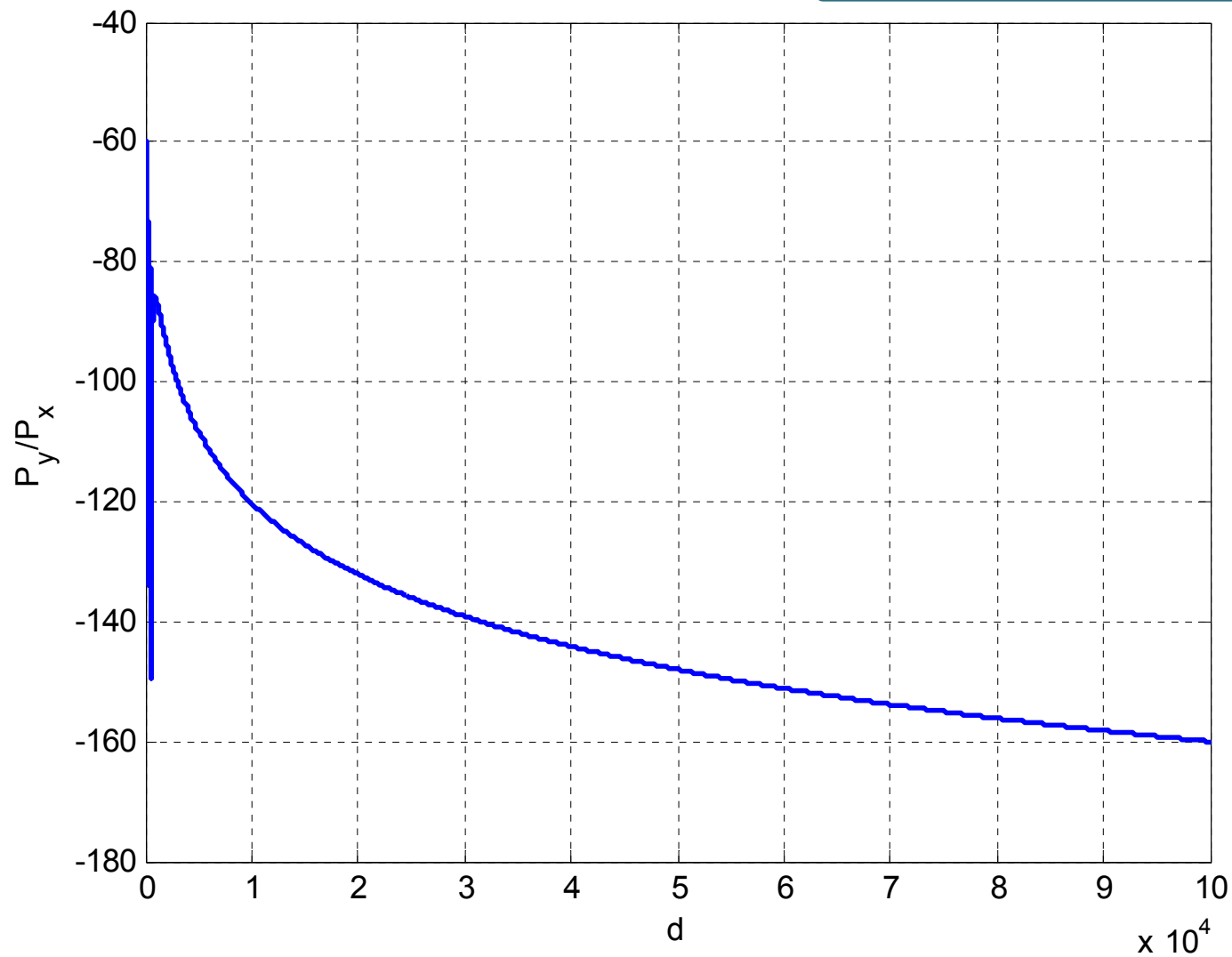
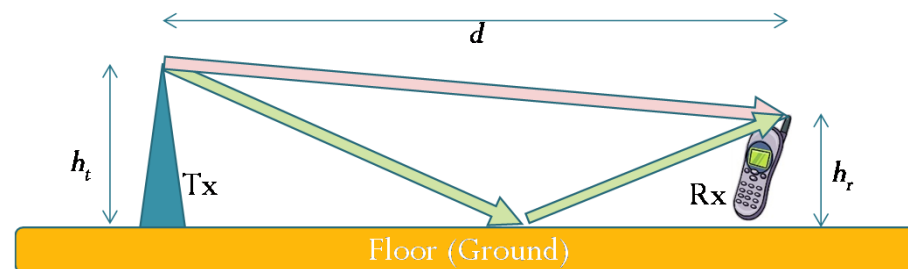
$$y(t) = \frac{\alpha}{r_1} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{r_1}{c}\right)\right) - \frac{\alpha}{r_2} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{r_2}{c}\right)\right)$$

$$\frac{P_y}{P_x} = \left| \frac{\alpha}{r_1} e^{-j2\pi f_c \frac{r_1}{c}} - \frac{\alpha}{r_2} e^{-j2\pi f_c \frac{r_2}{c}} \right|^2 = \left| \frac{\alpha}{r_1} - \frac{\alpha}{r_2} e^{-j2\pi f_c \frac{r_2 - r_1}{c}} \right|^2$$



Assume ground reflection coefficient = -1.

Ex. Two-Ray Model

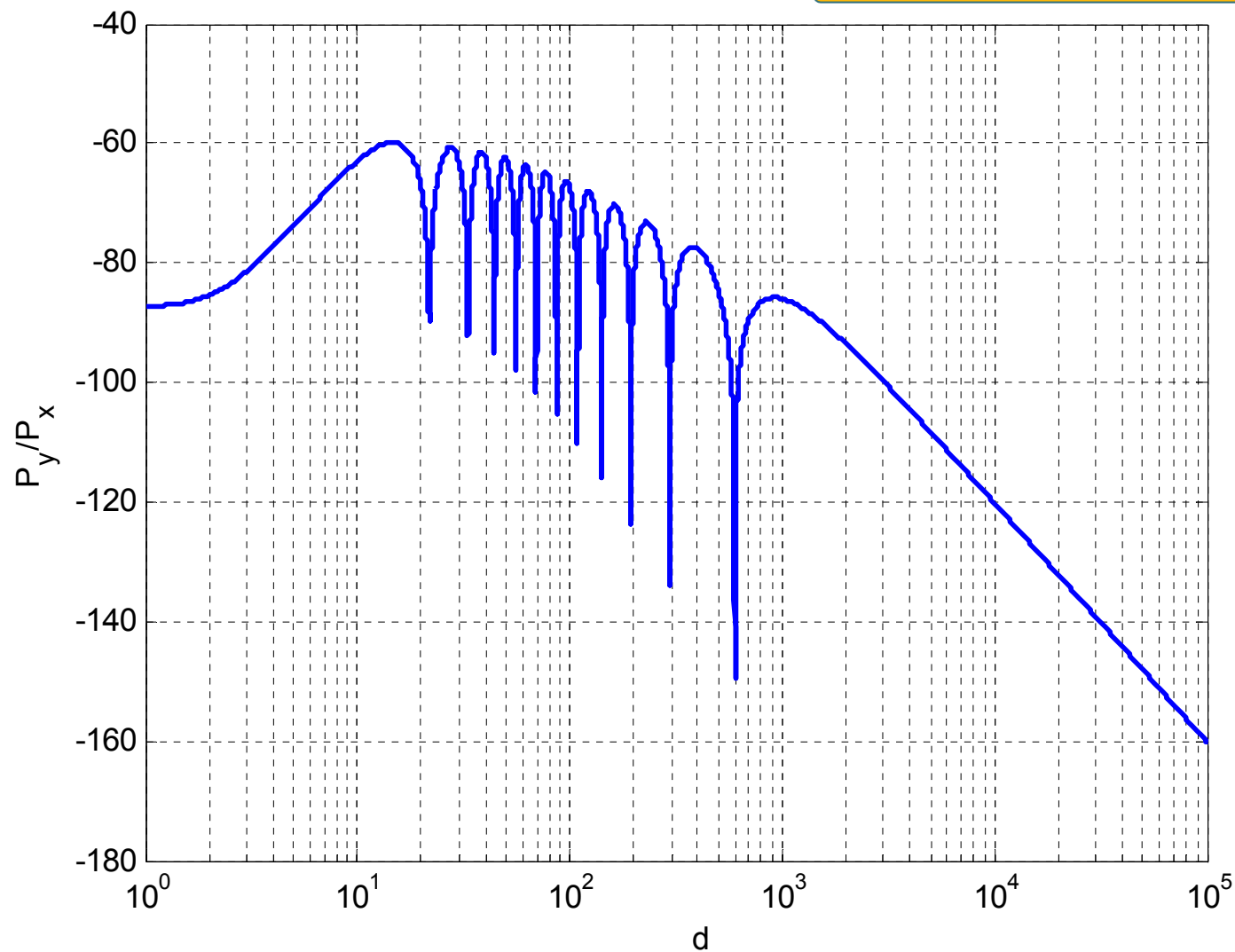
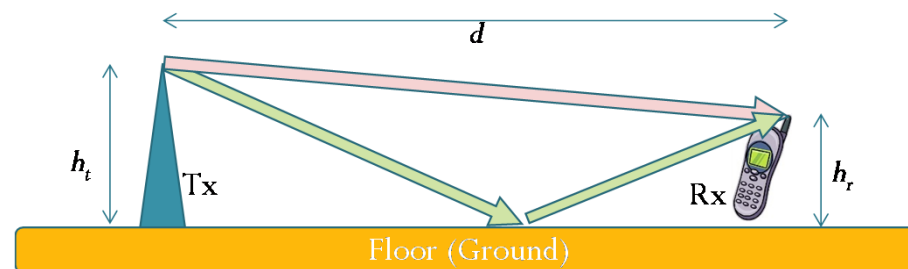


$$f = 900 \text{ MHz}$$

$$h_t = 50 \text{ m}$$

$$h_r = 2 \text{ m}$$

Ex. Two-Ray Model

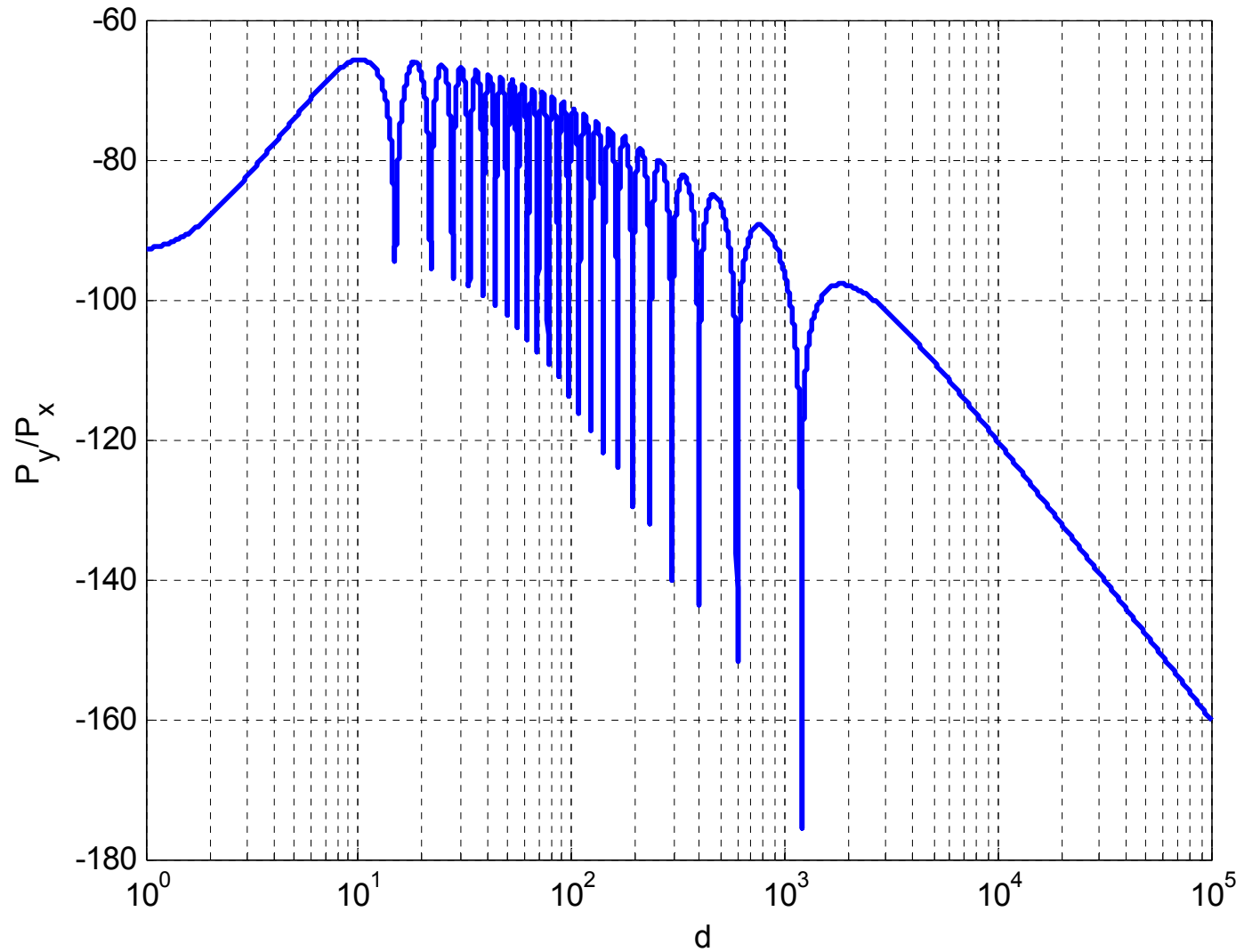
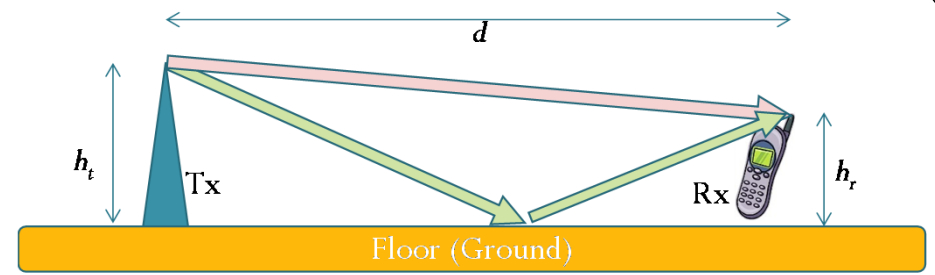


$$f = 900 \text{ MHz}$$

$$h_t = 50 \text{ m}$$

$$h_r = 2 \text{ m}$$

Ex. Two-Ray Model



$$f = 1800 \text{ MHz}$$

$$h_t = 50 \text{ m}$$

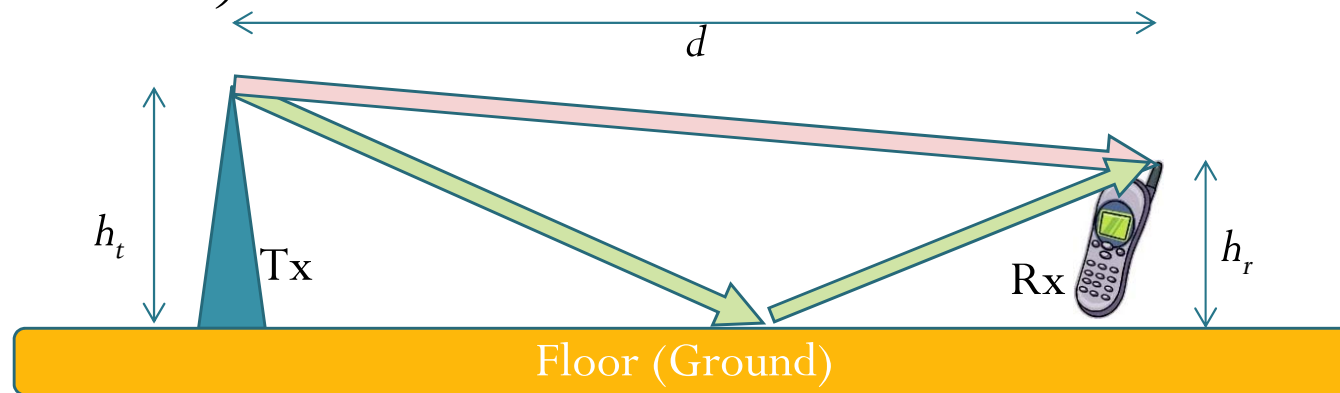
$$h_r = 2 \text{ m}$$

Ex. Two-Ray Model (Approximation)

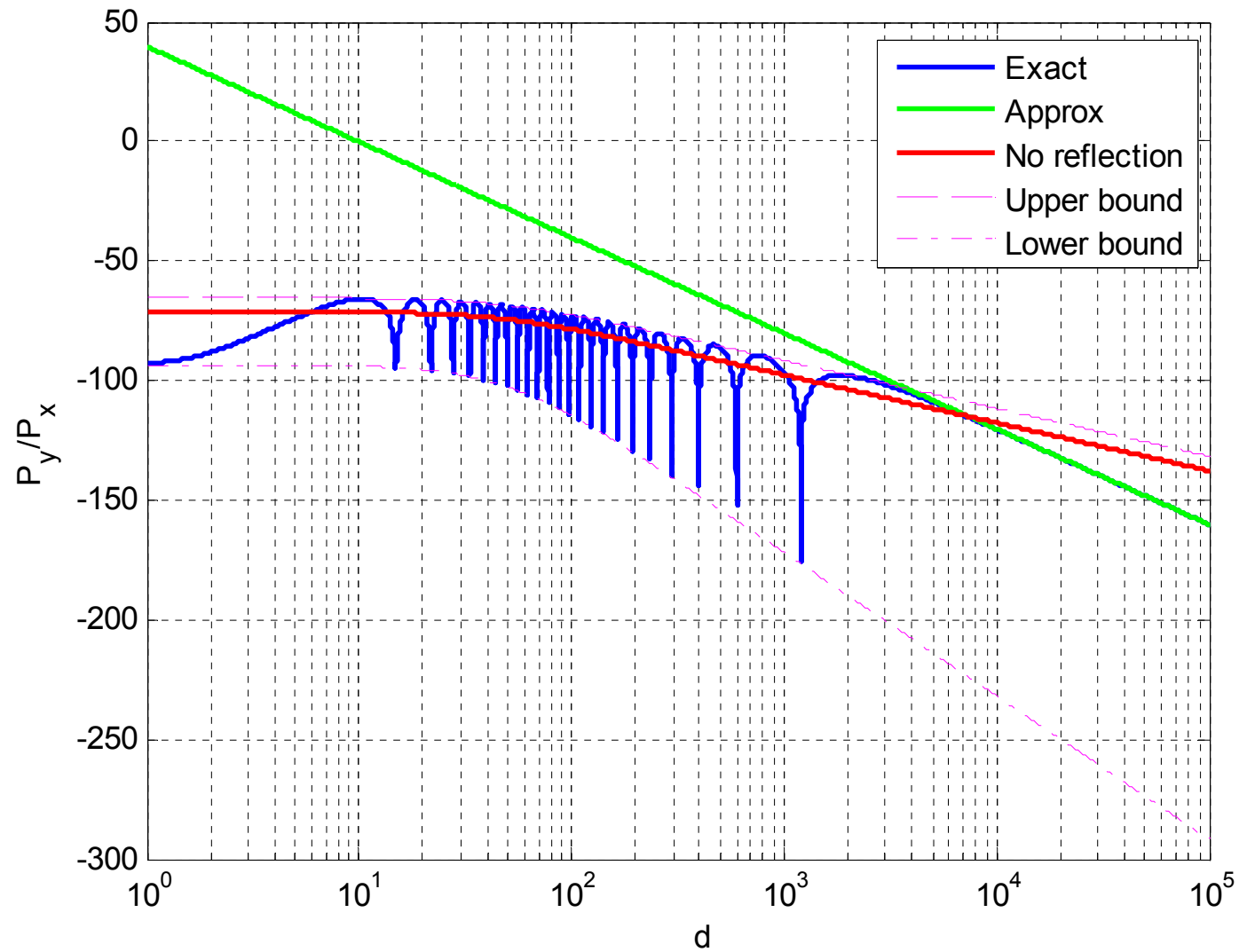
$$\frac{P_y}{P_x} \approx \left| \frac{\alpha}{r_1} - \frac{\alpha}{r_2} e^{-j2\pi \frac{2h_t h_r}{\lambda}} \right|^2 \approx \frac{\alpha}{d} \left| 1 - e^{-j2\pi \frac{2h_t h_r}{\lambda d}} \right|^2 \quad d \gg h_t, h_r$$

$$\approx \left(\frac{\alpha}{d} \right)^2 \left| 1 - \left(1 - j2\pi \frac{2h_t h_r}{\lambda d} \right) \right|^2 = \frac{\alpha^2}{d^2} \left| j2\pi \frac{2h_t h_r}{\lambda d} \right|^2 = \left(\frac{4\pi\alpha h_t h_r}{\lambda d^2} \right)^2$$

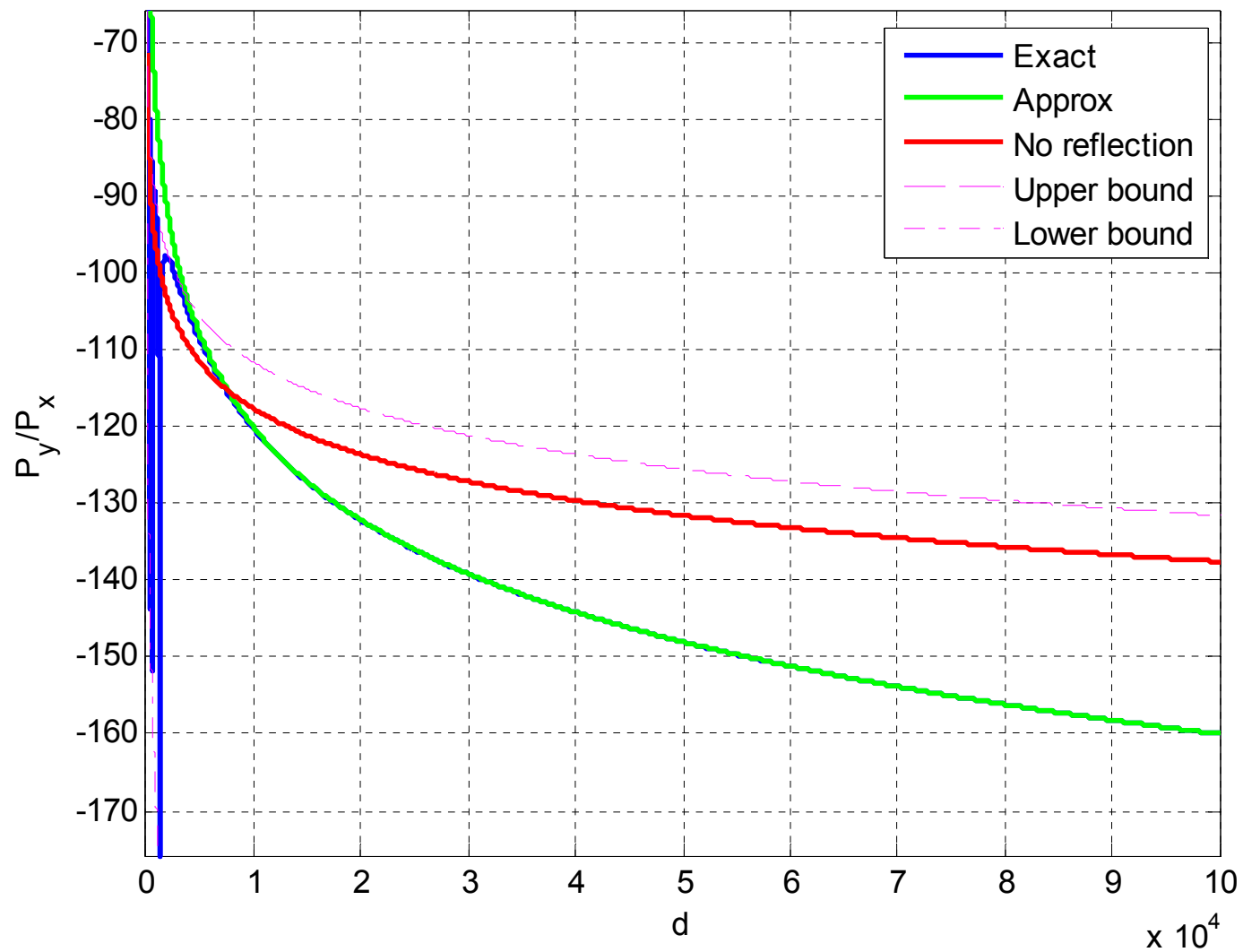
$$= \left(\frac{\sqrt{G_{Tx} G_{Rx}} h_t h_r}{d^2} \right)^2 \propto \frac{1}{d^4}$$



Ex. Two-Ray Model



Ex. Two-Ray Model

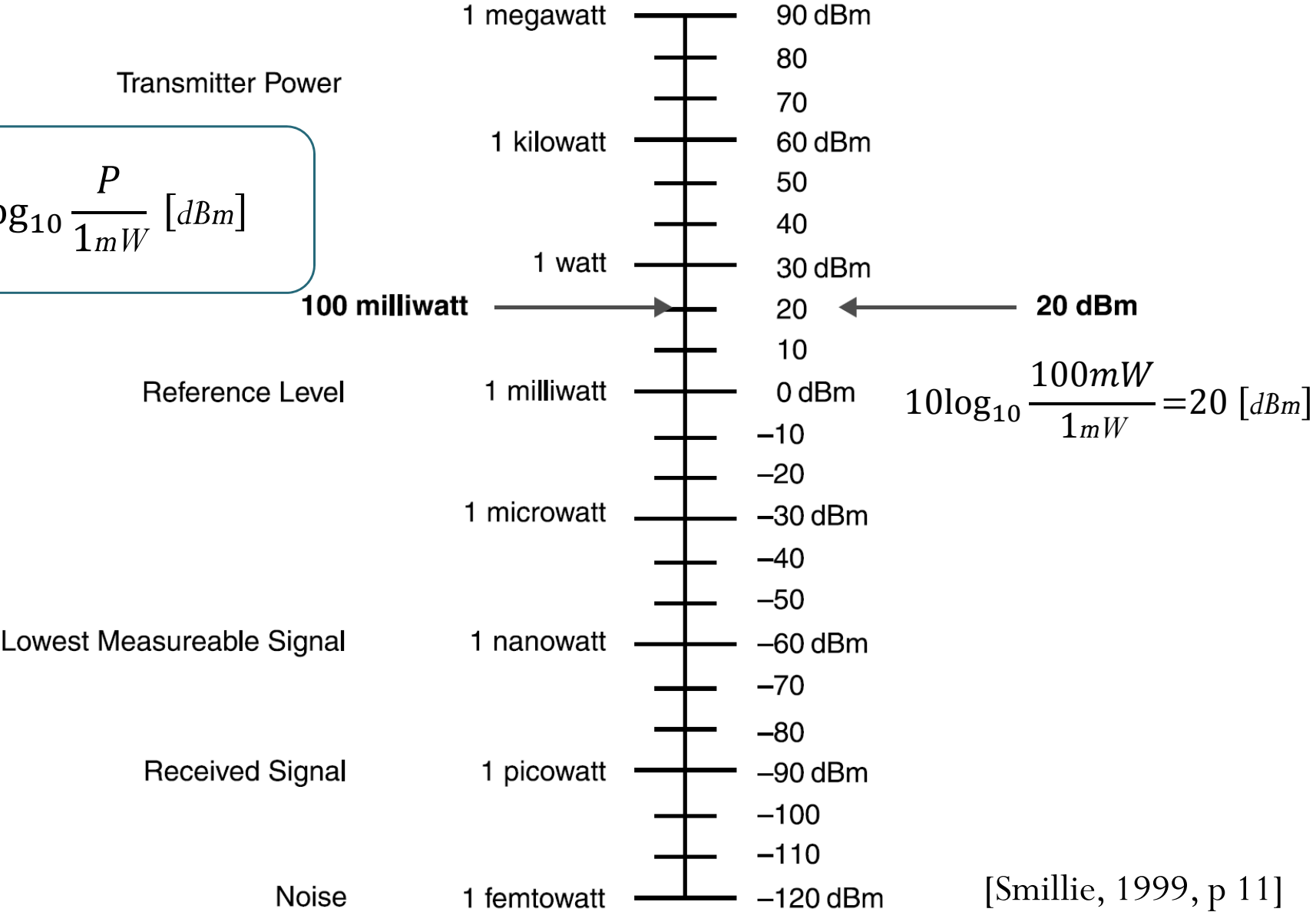


dBm

- The range of RF power that must be measured in cellular phones and wireless data transmission equipment varies from
 - hundreds of watts in base station transmitters to
 - picowatts in receivers.
- For calculations to be made, all powers must be expressed in the same power units, which is usually **milliwatts**.
 - A transmitter power of 100 W is therefore expressed as 100,000mW. A received power level of 1 pW is therefore expressed as 0.000000001mW.
- Making power calculations using decimal arithmetic is therefore complicated.
- To solve this problem, the dBm system is used.

Range of RF Power in Watts and dBm

$$P [W] = 10 \log_{10} \frac{P}{1mW} [dBm]$$



dB and dBm

- The decibel scale expresses factors or ratios logarithmically.
- Unitless dB value
 - Represent power ratio: $10\log_{10} \frac{P_2}{P_1}$
- dB value with a unit

- Represent the signal power itself:

$$P[\text{dBW}] = 10\log_{10} \frac{P}{1 \text{ W}}, \quad P[\text{dBm}] = 10\log_{10} \frac{P}{1 \text{ mW}}$$

- Note that $P[\text{dBm}] = P[\text{dBW}] + 30$

Remark

- Adding dB values corresponds to multiplying the underlying factors, which means multiplying the units if they are present.
- It is therefore appropriate to add unitless dB values to a dB value with a unit (such as dBm)
 - The result is still referred to that unit.
 - Ex: $17 \text{ dBm} + 13 \text{ dB} - 6 \text{ dB} = 24 \text{ dBm}$
 - Correspond to $50 \text{ mW} \times 20 / 4 = 250 \text{ mW}$.

Doppler Shift: 1D Move

- At the transmitter, suppose we have

$$\sqrt{2P_t} \cos(2\pi f_c t + \phi)$$

- At distance r (far enough), we have **Time to travel a distance of r**

$$\frac{\alpha}{r} \sqrt{2P_t} \cos\left(2\pi f_c \left(t - \frac{r}{c}\right) + \phi\right)$$

- If moving, r becomes $r(t)$.
- If moving **away** at a constant velocity v , then $r(t) = r_0 + vt$.

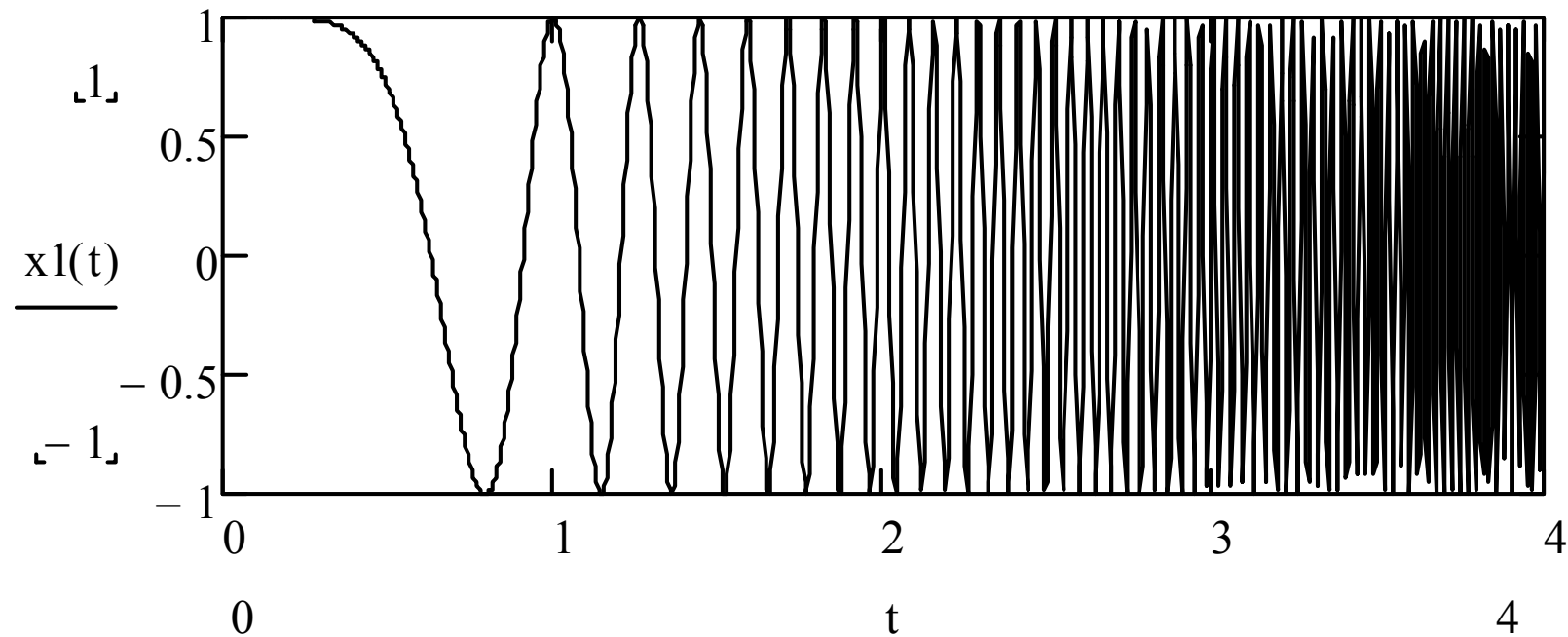
$$\frac{\alpha}{r(t)} \cos\left(2\pi f_c \left(t - \frac{r_0 + vt}{c}\right) + \phi\right) = \frac{\alpha}{r(t)} \cos\left(2\pi \left(f_c - f_c \frac{v}{c}\right) t - 2\pi f_c \frac{r_0}{c} + \phi\right)$$

Frequency shift

$$\Delta f = \frac{v}{\lambda}$$

Instantaneous Frequency

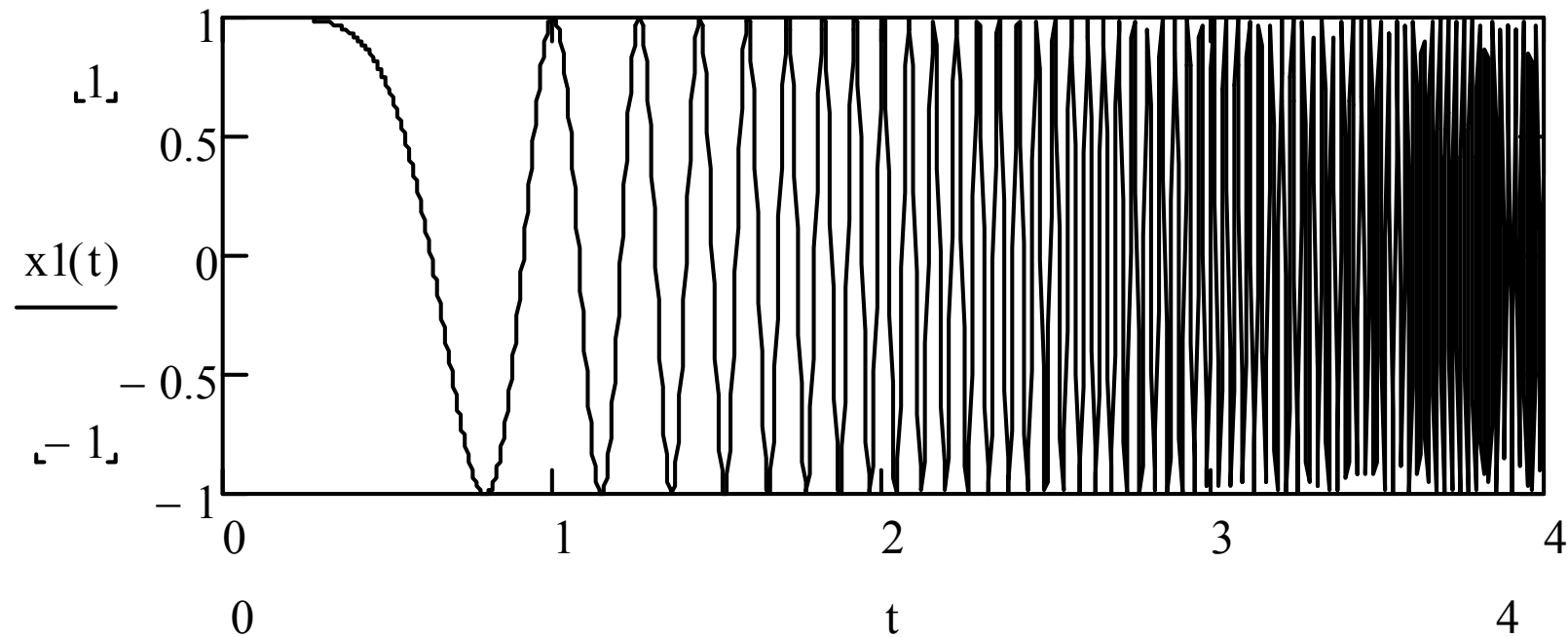
$$x_1(t) = \cos(2\pi t^2 t)$$



At $t = 2$, frequency = ?

Instantaneous Frequency

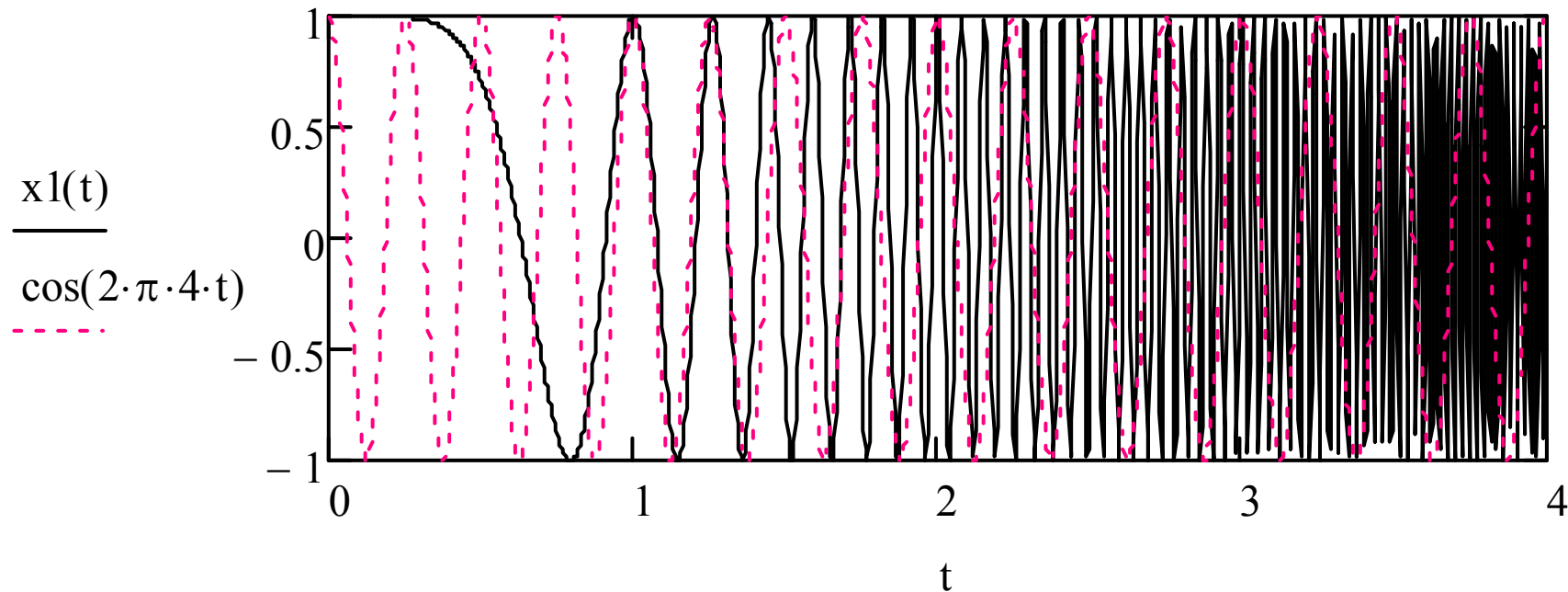
$$x_1(t) = \cos(2\pi t^2 t)$$



$\cos(2\pi ft)$ \longrightarrow At $t = 2$, $f = t^2 = 4$ Hz?

Instantaneous Frequency

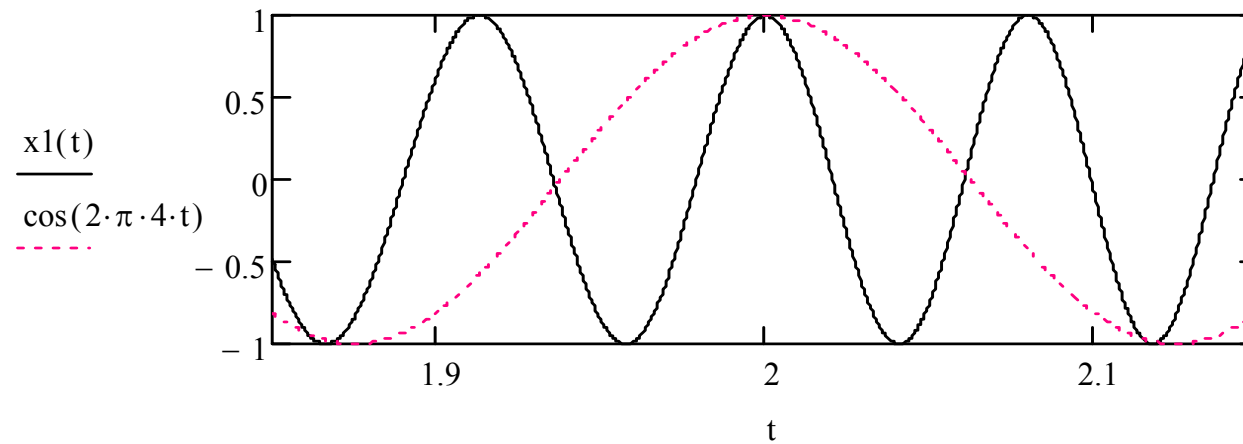
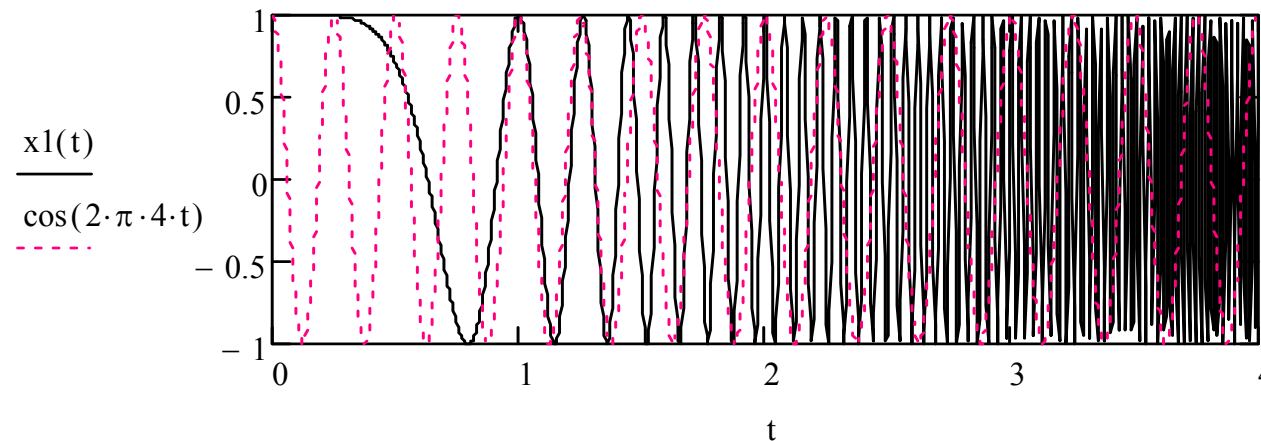
$$x_1(t) = \cos(2\pi t^2 t)$$



$\cos(2\pi ft)$ \longrightarrow At $t = 2$, $f = t^2 = 4$ Hz?

Instantaneous Frequency

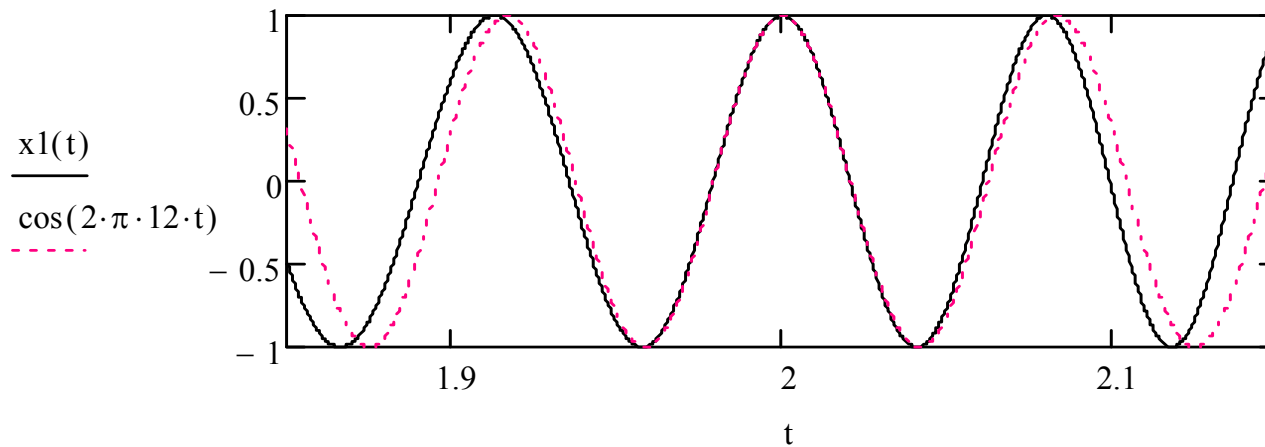
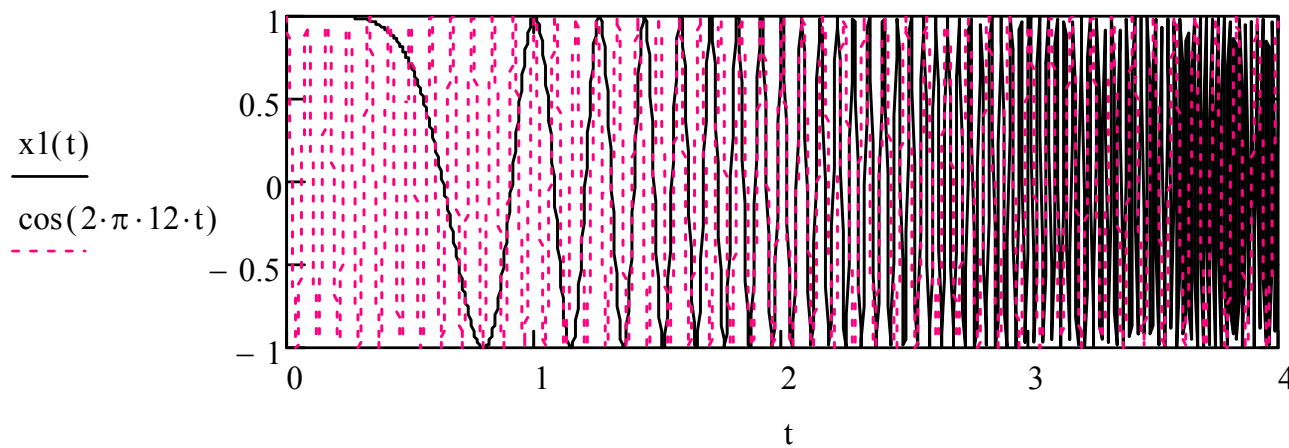
$$x_1(t) = \cos(2\pi t^2 t)$$



4 Hz is too low!!!

Instantaneous Frequency

$$x_1(t) = \cos(2\pi t^2 t)$$



12 Hz?

Review: Instantaneous Frequency

For a generalized sinusoid signal

$$A \cos(\theta(t)),$$

the **instantaneous frequency** at time t is given by

$$f(t) = \frac{1}{2\pi} \frac{d}{dt} \theta(t).$$

When $\theta(t) = 2\pi f_c \left(t - \frac{r(t)}{c} \right) + \phi,$

$$f(t) = \frac{1}{2\pi} \frac{d}{dt} \theta(t) = f_c - \frac{f_c}{c} \frac{d}{dt} r(t) = f_c - \frac{1}{\lambda} \frac{d}{dt} r(t)$$

Frequency shift

Big Picture

Transmission impairments in cellular systems

Physics of radio propagation

Attenuation (Path Loss)

Shadowing

Doppler shift

Inter-symbol interference (ISI)

Flat fading

Frequency-selective fading

Extraneous signals

Co-channel interference

Adjacent channel interference

Impulse noise

White noise

Transmitting and receiving equipment

White noise

Nonlinear distortion

Frequency and phase offset

Timing errors